Centro de Investigação em Matemática e Aplicações

Departamento de Matemática

Programa de Doutoramento em Matemática

Seminário

31 de Outubro de 2018

CLAV-Room nº138- 10h

Measure disintegration on product spaces and a generalization of Fubini's Theorem

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Abstract

Let μ be a measure on a product of polish spaces $X \times Y$. We will revisit the disintegration of measures, a tool to convert the integrals against μ into iterated integrals. Denoting by μ_X the marginal measure of μ over X, we show the existence (μ_X-almost everywhere) of a family of measures on Y, indexed by the points on X such that any integral against μ (on $X \times Y$) can be realized by integrating one variable at a time, first on Y with respect to this family, followed by integration on X against μ_X. We can reverse the order of integration by interchanging the roles of X and Y in the process above. This tool can be applied to certain problems in Optimal

Transportation, where a cost functional is optimized over a set of admissible measures on a product space.

Keywords: Measure disintegration, Fubini's theorem, Optimal Transportation.

Acknowledgements

This talk has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA), through the grant UID-MAT-04674-2013 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

Projecto UID/MAT/04674/2013

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